Geometry Notes

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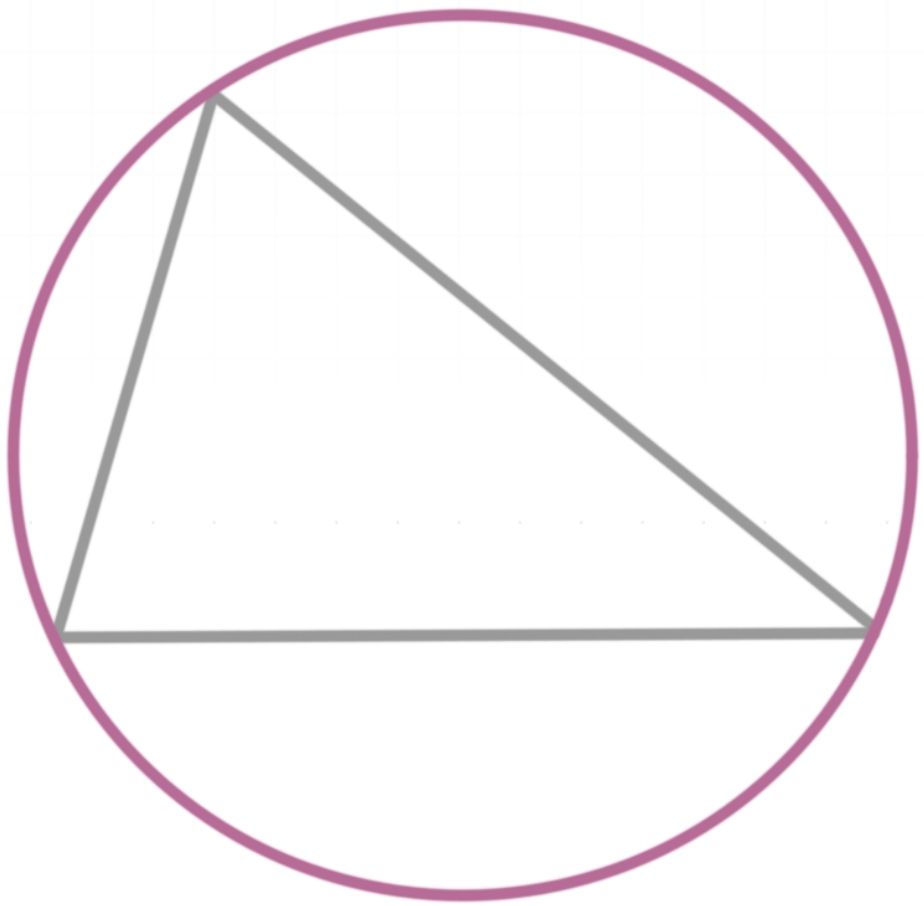
MATH

Circumscribed and inscribed circles of a triangle

In this lesson we'll look at circumscribed and inscribed circles and the special relationships that form from these geometric ideas.

Circumscribed circles

When a circle circumscribes a triangle, the triangle is inside the circle and the triangle touches the circle with each vertex.



You use the perpendicular bisectors of each side of the triangle to find the the center of the circle that will circumscribe the triangle. So for example, given $\triangle GHI$,

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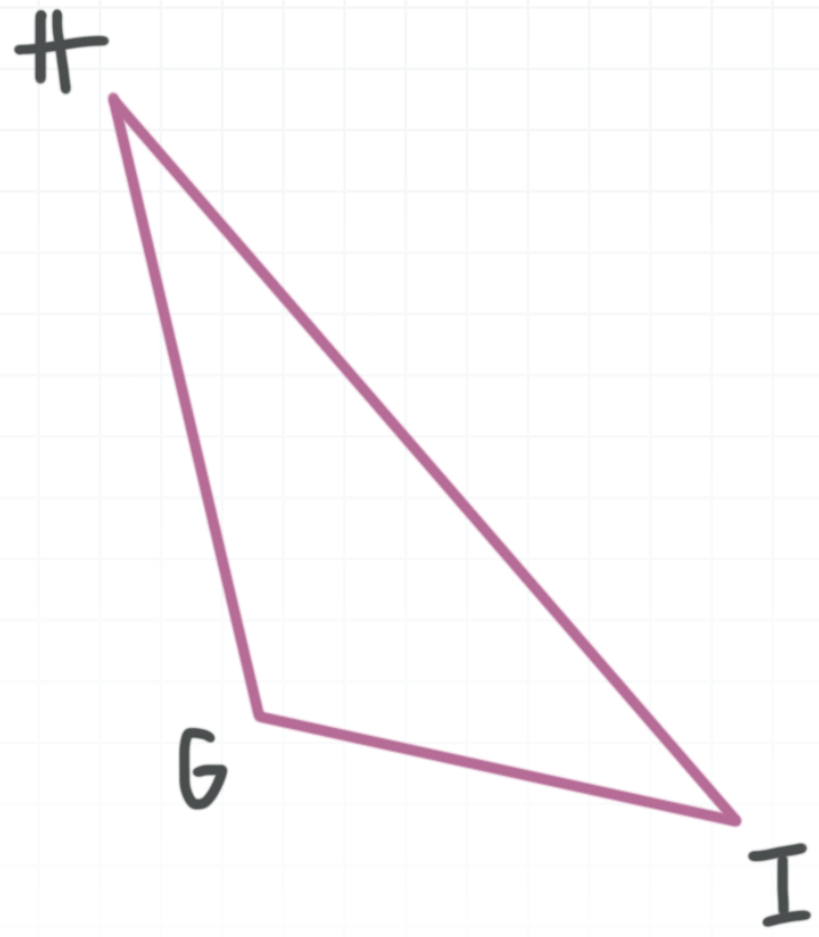
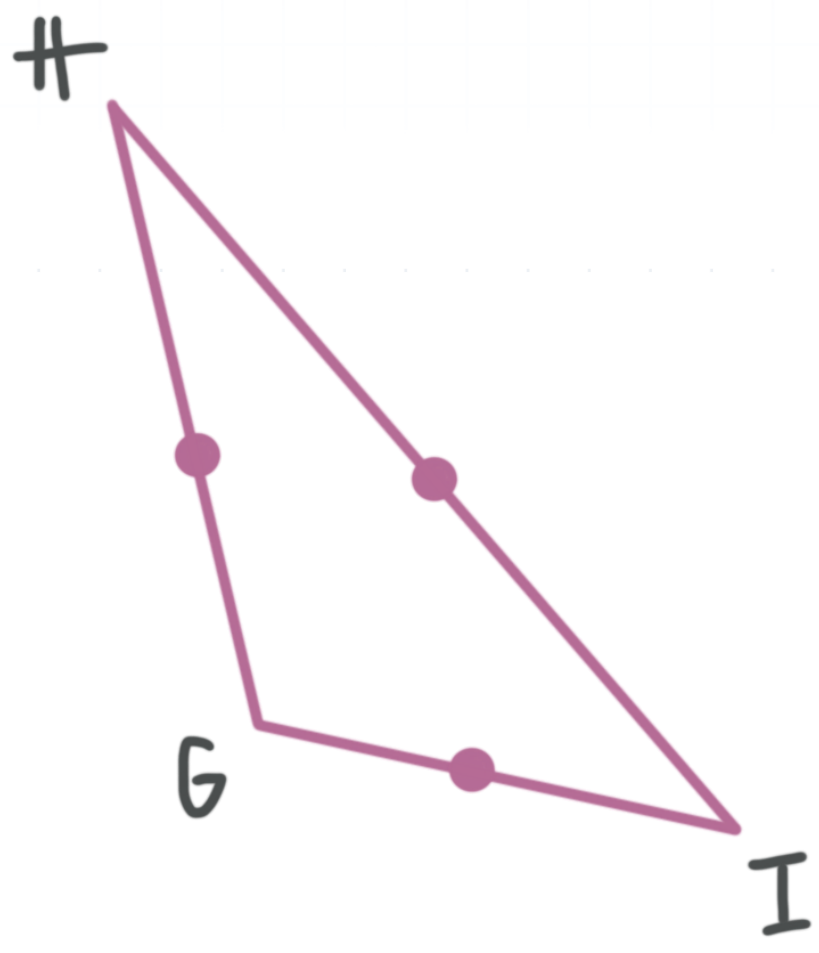


find the midpoint of each side.



Find the perpendicular bisector through each midpoint.

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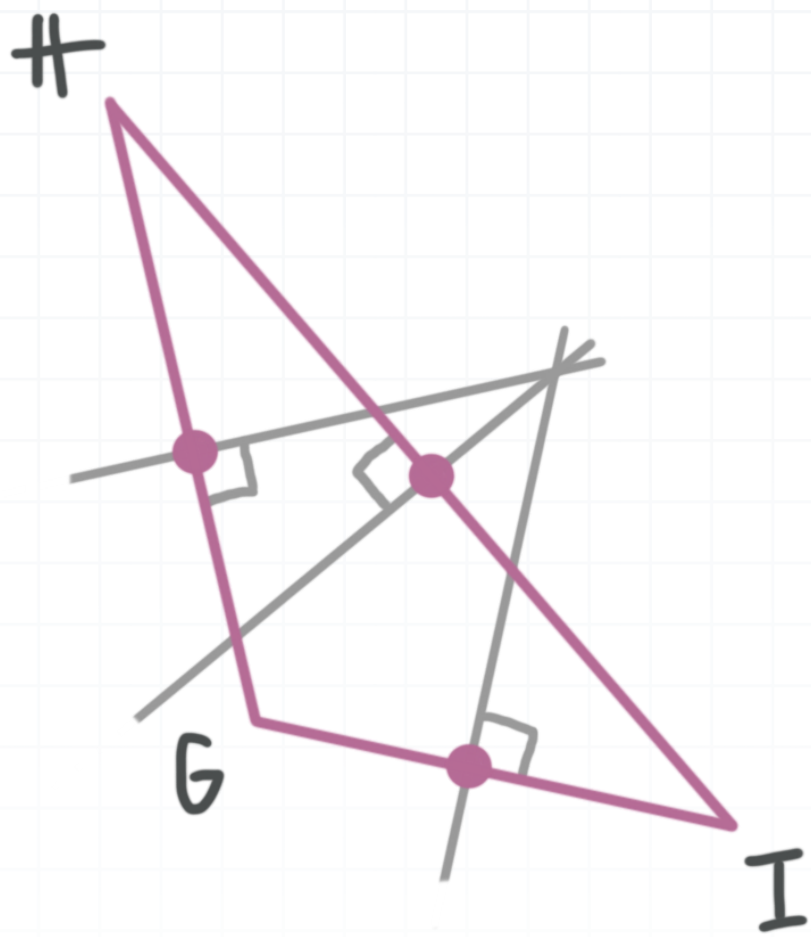
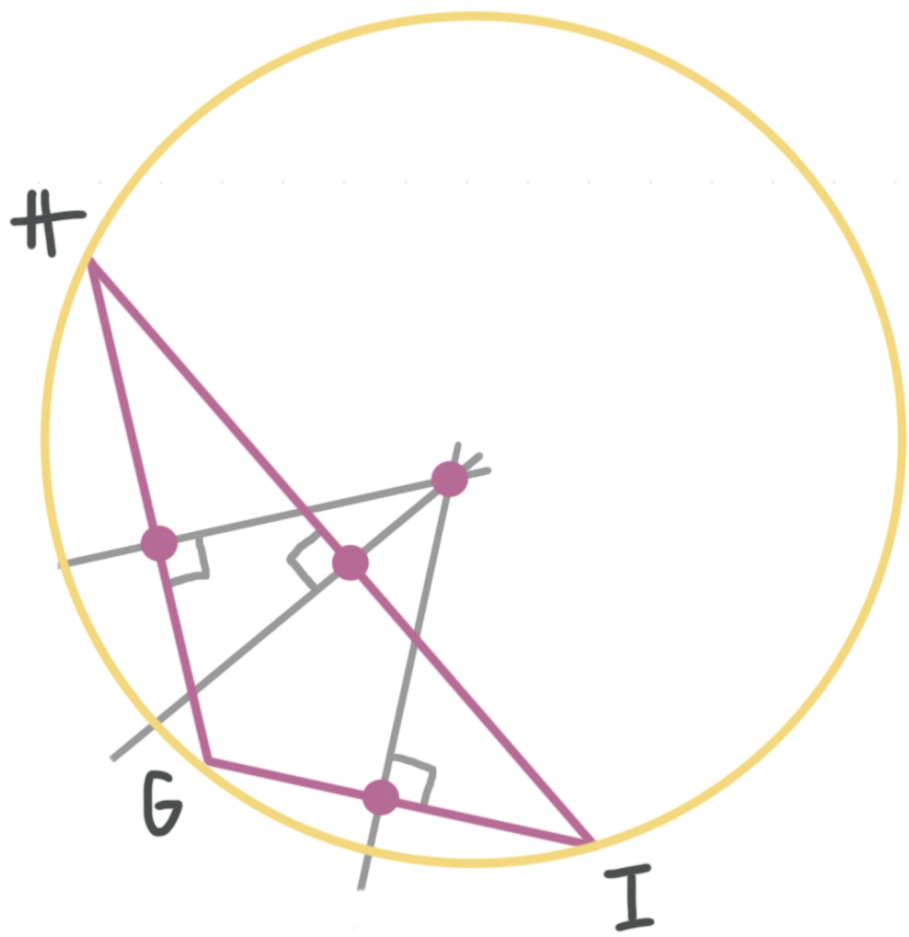


The point where the perpendicular bisectors intersect is the center of the circle.



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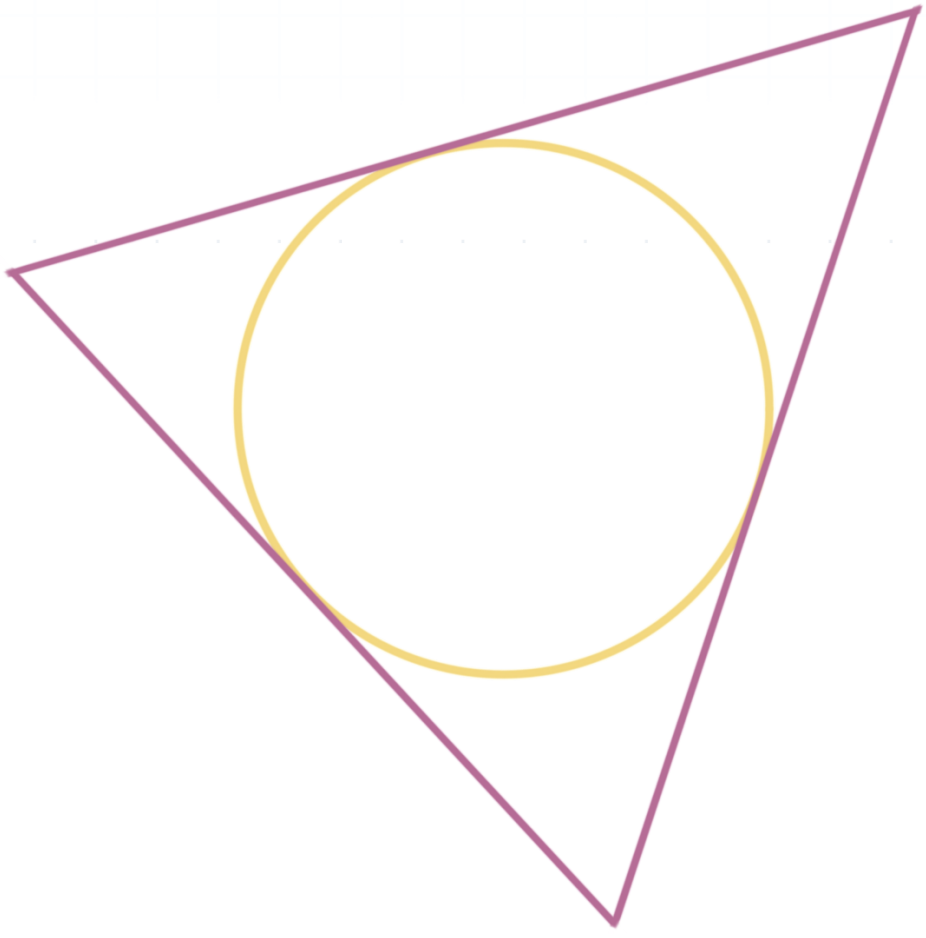
The center point of the circumscribed circle is called the "**circumcenter**."

- For an acute triangle, the circumcenter is inside the triangle.
- For a right triangle, the circumcenter is on the side opposite right angle.
- For an obtuse triangle, the circumcenter is outside the triangle.

Inscribed circles

When a circle inscribes a triangle, the triangle is outside of the circle and the circle touches the sides of the triangle at one point on each side. The sides of the triangle are tangent to the circle.



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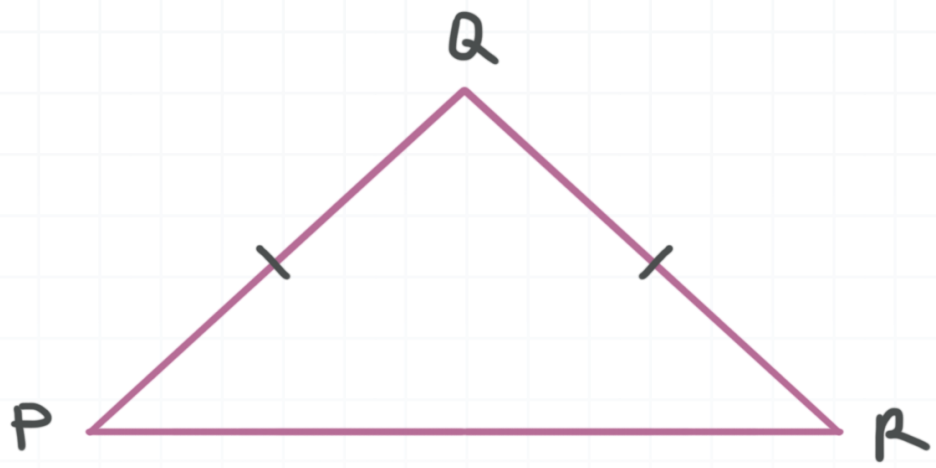
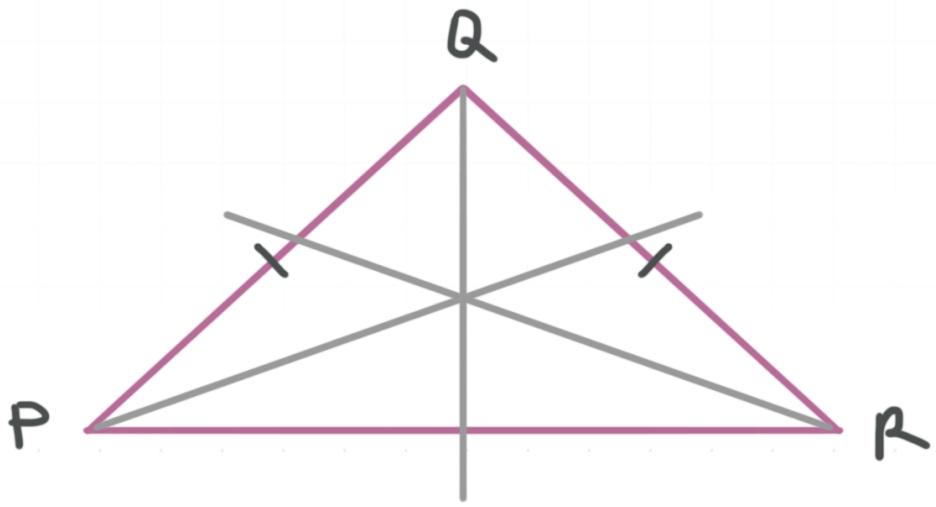
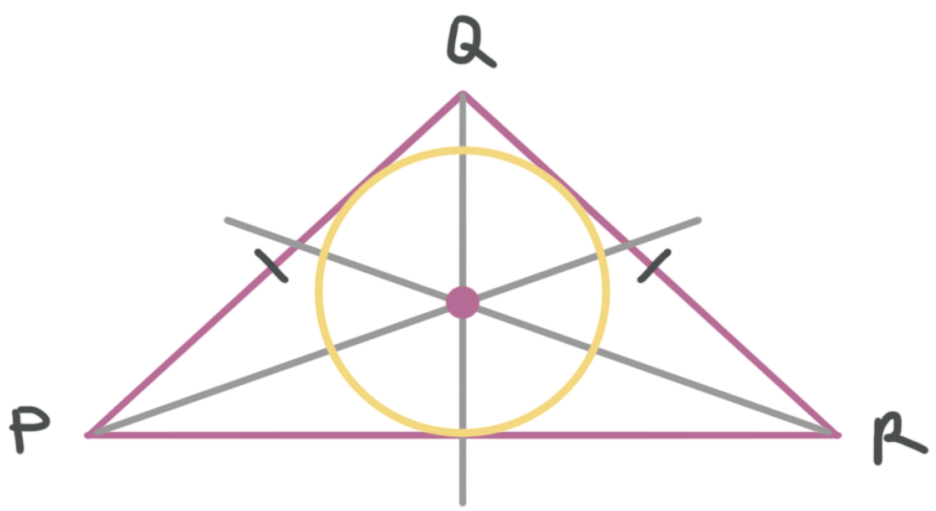
To drawing an inscribed circle inside an isosceles triangle, use the angle bisectors of each side to find the center of the circle that's inscribed in the triangle. For example, given $\triangle PQR$,



draw in the angle bisectors.



The intersection of the angle bisectors is the center of the inscribed circle.



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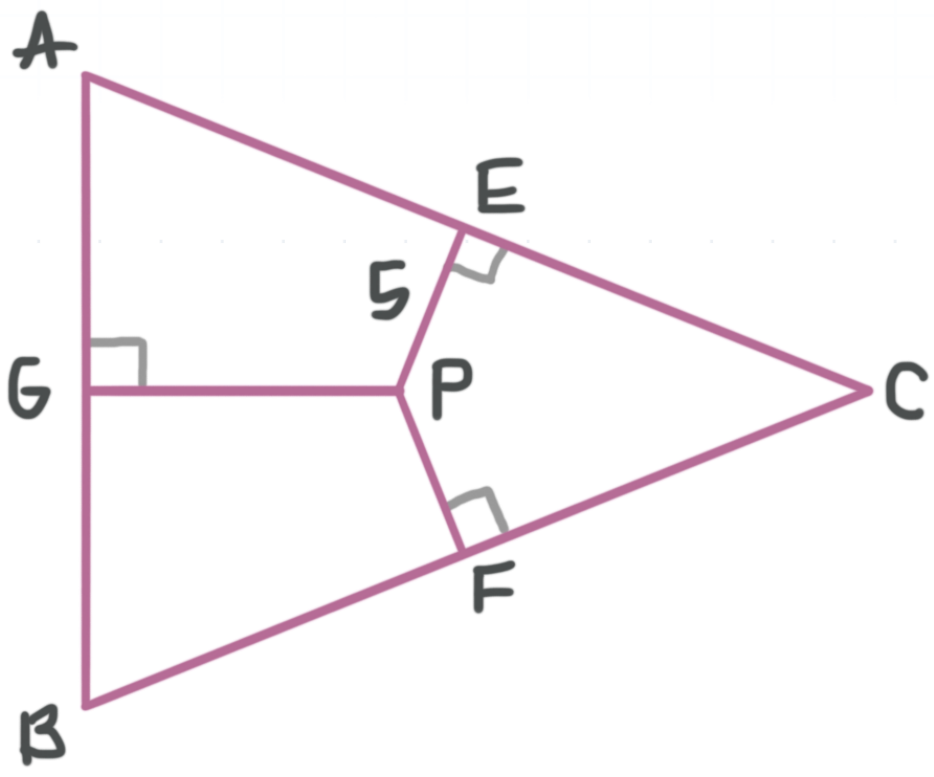
Remember that each side of the triangle is tangent to the circle, so if you draw a radius from the center of the circle to the point where the circle touches the edge of the triangle, the radius will form a right angle with the edge of the triangle.

The center point of the inscribed circle is called the "**incenter**." The incenter will always be inside the triangle.

Let's use what we know about these constructions to solve a few problems.

Example

$\overline{GP}$, $\overline{EP}$, and $\overline{FP}$ are the perpendicular bisectors of $\triangle ABC$, and $AC = 24$ units. What is the measure of the radius of the circle that circumscribes $\triangle ABC$?



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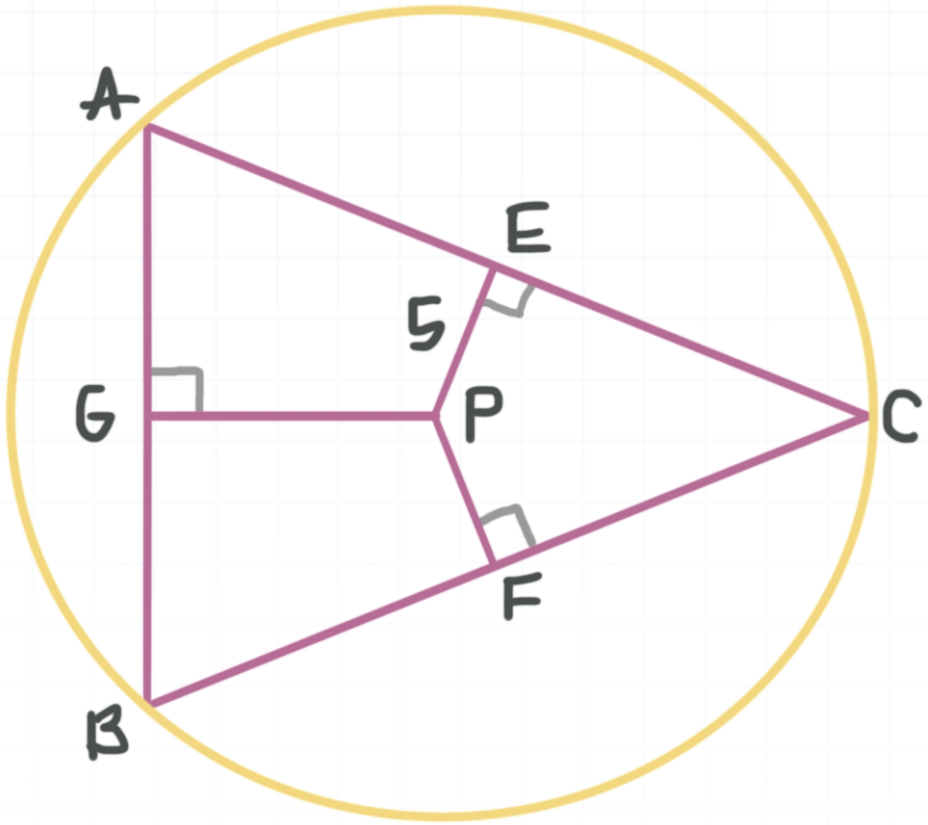
Point P is the circumcenter of the circle that circumscribes $\triangle ABC$ because it's where the perpendicular bisectors of the triangle intersect. We can draw $\bigcirc P$.



We also know that $AC = 24$ units, and since $\overline{EP}$ is a perpendicular bisector of $\overline{AC}$, point E is the midpoint. Therefore,

$$EC = \frac{1}{2}AC = \frac{1}{2}(24) = 12$$

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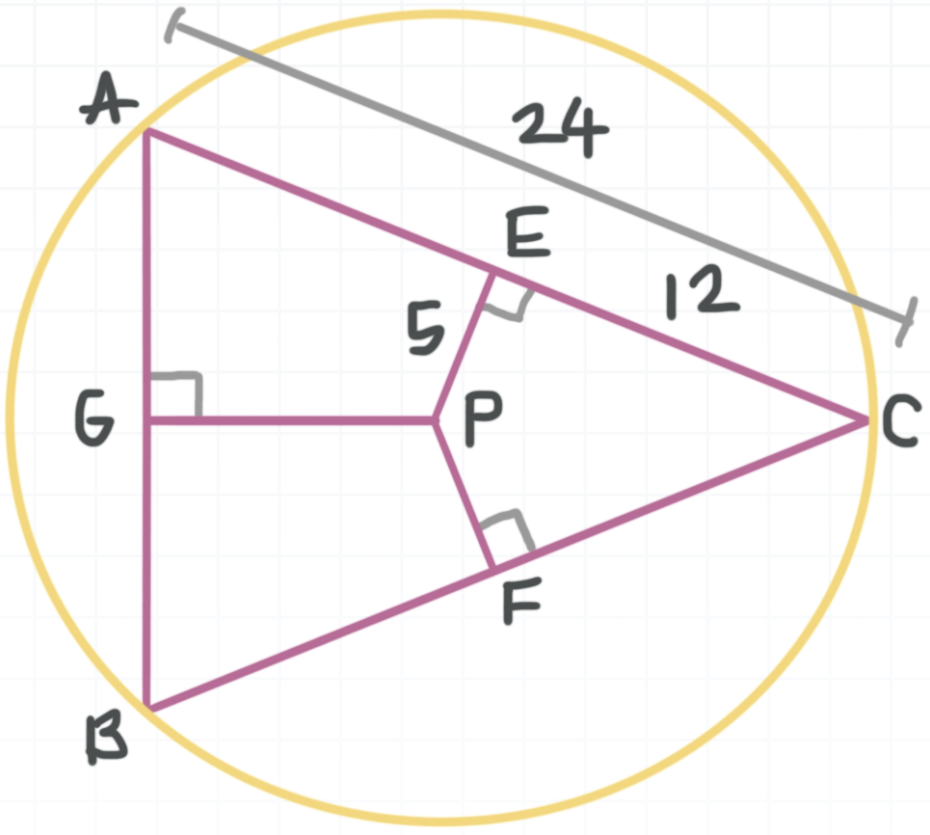
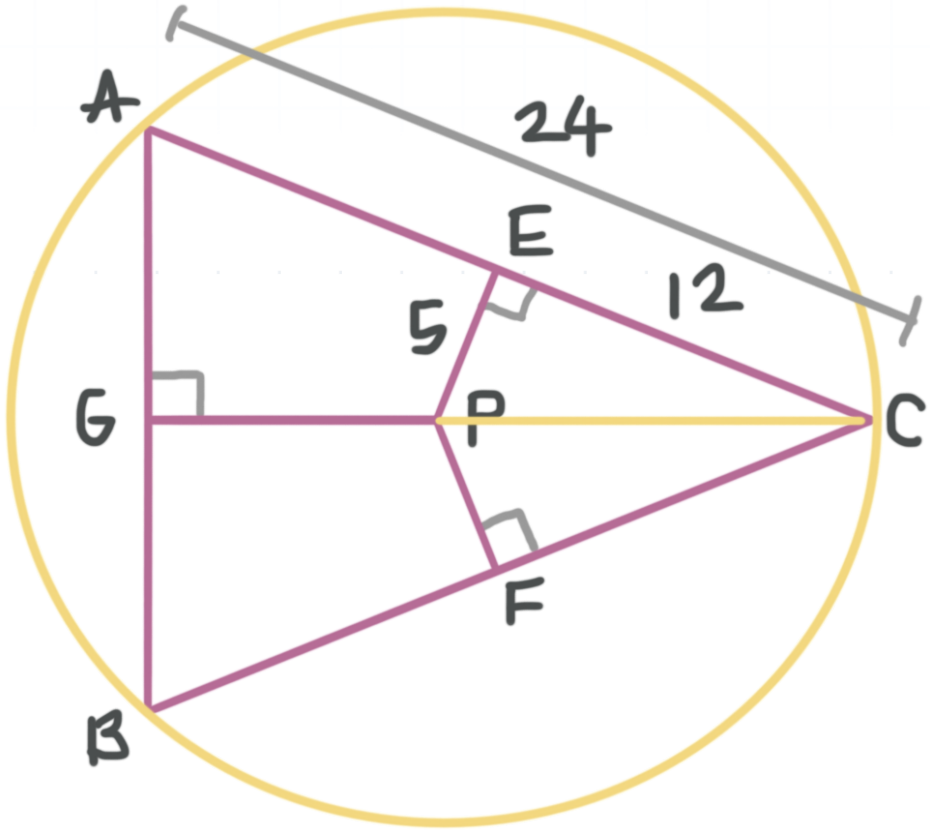


Now we can draw the radius from point P, the center of the circle, to point C, a point on its circumference.



We can use right $\triangle PEC$ and the Pythagorean theorem to solve for the length of radius $\overline{PC}$.

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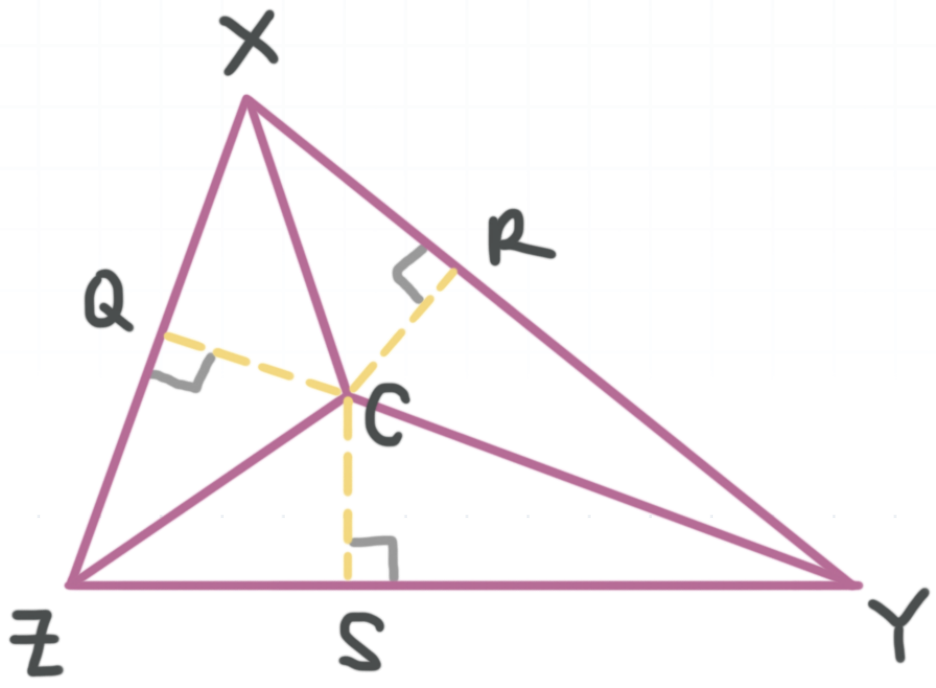
$$5^2 + 12^2 = (PC)^2$$

$$PC = 13$$

Let's try a different problem.

Example

If $CQ = 2x - 7$ and $CR = x + 5$, what is the measure of CS, given that $\overline{XC}$, $\overline{YC}$, and $\overline{ZC}$ are angle bisectors of $\triangle XYZ$.



Because $\overline{XC}$, $\overline{YC}$, and $\overline{ZC}$ are angle bisectors of $\triangle XYZ$, C is the incenter of the triangle. The circle with center C will be tangent to each side of the triangle at the point of intersection.

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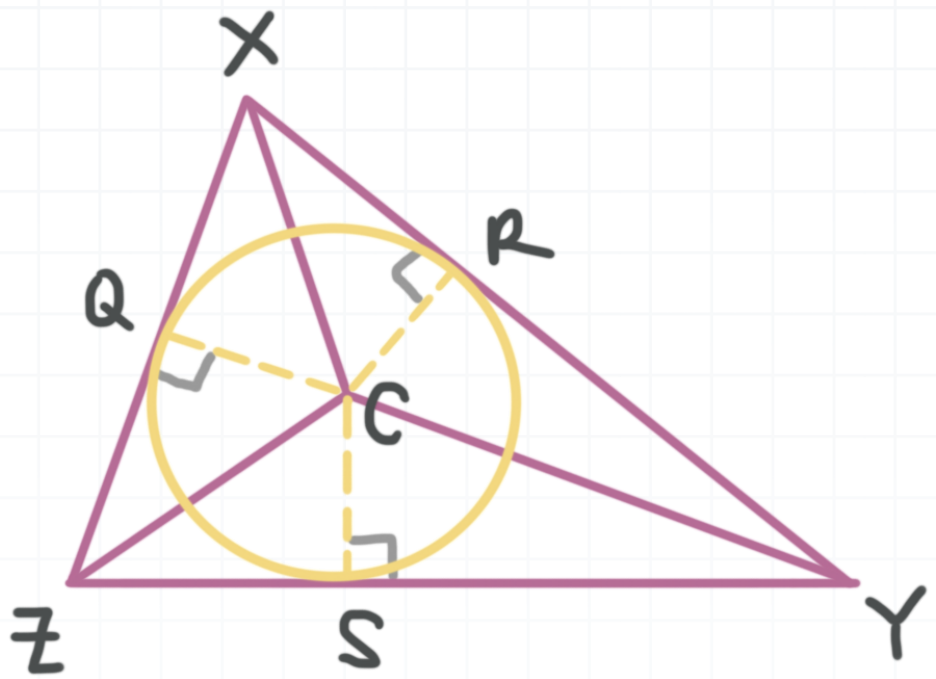

$\overline{CQ}$, $\overline{CR}$, and $\overline{CS}$ are all radii of circle C, so they're all equal in length.

$$CQ = CR = CS$$

We need to find the length of a radius. We know $CQ = 2x - 7$ and $CR = x + 5$, so

$$CQ = CR$$

$$2x - 7 = x + 5$$

$$x = 12$$

Therefore,

$$CQ = CR = CS = x + 5 = 12 + 5 = 17$$

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